AVIRAL
JEE | NEET | FOUNDATIONS

WEEKLY TEST TYJ TEST - 28 B
SOLUTION Date 24-11-2019

[PHYSICS]

1 Rate of transmission of heat by conduction is given by,

$\frac{dQ}{dt} = \frac{KA\Delta\theta}{l}$, for both rods K, A and $\Delta\theta$ are same.

where, symbols have their usual meaning.

$\therefore \qquad \frac{dQ}{dt} \propto \frac{1}{l}$

So, $\frac{(dQ/dt)_{\text{semicircular}}}{(dQ-dt)_{\text{straight}}} = \frac{l_{\text{straight}}}{l_{\text{semicircular}}} = \frac{2r}{\pi r} = \frac{2}{\pi}$

2. Thermal resistance in configuration I,

$$R_{\text{I}} = R_1 + R_2 = \left(\frac{l}{KA}\right) + \left(\frac{l}{2KA}\right) = \frac{3}{2}\left(\frac{l}{KA}\right)$$

Thermal resistance in configuration II,

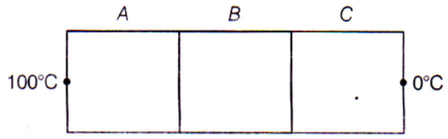
$$\frac{1}{R_{\text{II}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{KA}{l} + \frac{2KA}{l}$$

or $\qquad R_{\text{II}} = \frac{l}{3KA} = \frac{R_1}{4.5}$

Since, thermal resistance R_{II} is 4.5 times less than thermal resistance R_{I}

$\therefore \qquad t_{\text{II}} = \frac{t_1}{4.5} = \frac{9}{4.5}\text{ s} = 2\text{s}$

3 Let R be the thermal conductivity of conductor A, then thermal conductivity of conductor $B = \frac{R}{2}$.



and thermal conductivity of conductor $C = 2R$

$\therefore$ Heat current, $H = \frac{100° - 0°}{R + \frac{R}{2} + 2R} = \frac{200}{7R}$

If T' be the temperature of the junction of A and B, then

$$H = \frac{100 - T'}{R} \text{ or } \frac{200}{7R} = \frac{100 - T'}{R}$$

or $\qquad T' = \frac{500}{7} = 71°\text{C}$

4. According to question,

A	B	C

• $K_B = \frac{K_A}{2} \Rightarrow K_B = 3K_C$

$$K_C = \frac{K_A}{6}$$

$$\Rightarrow \quad \frac{l}{K_S} = \frac{l_1}{K_A} + \frac{l_2}{K_B} + \frac{l_3}{K_C}$$

$$\frac{3l}{K_S} = \frac{9l}{K_A} \Rightarrow K_S = \frac{K_A}{3}$$

5. As we know that rate of flow of heat is given by,

$$\frac{dQ}{dt} = \frac{KA\Delta T}{x}$$

$$\frac{1.56\times10^5}{3600} = \frac{K\times2\times20}{12\times10^{-2}}$$

$$K = \frac{1.56\times10^5\times12\times10^{-2}}{3600\times2\times20} = \frac{1.56}{12} = 0.13$$

6. We know that, $\frac{dQ}{dt} = KA\frac{d\theta}{dx}$

In steady state, flow of heat,

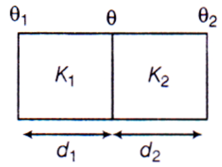
$$d\theta = \frac{dQ}{dt}\cdot\frac{1}{KA}dx$$

$$\Rightarrow \quad \theta_H - \theta = K'x \Rightarrow \theta = \theta_H - K'x$$

Equation, $\theta = \theta_H - K'x$ represents a straight line.

7. For first slab,

heat current, $H_1 = \frac{K_1(\theta_1-\theta)A}{d_1}$



For second slab,

heat current, $H_2 = \frac{K_2(\theta-\theta_2)A}{d_2}$

As slabs are in series

$$H_1 = H_2$$

$$\therefore \quad \frac{K_1(\theta_1-\theta)A}{d_1} = \frac{K_2(\theta-\theta_2)A}{d_2}$$

$$\Rightarrow \quad \theta = \frac{K_1\theta_1 d_2 + K_2\theta_2 d_1}{K_2 d_1 + K_1 d_2}$$

8. Heat current, $\frac{Q}{t} \propto \frac{r^2}{l}$, from the given options, option (b) has higher value of $\frac{r^2}{l}$. Hence, $r = 2r_o$ and $l = l_o$.

9. Temperature of interface,

$$\theta = \frac{K_1\theta_1 l_2 + K_2\theta_2 l_1}{K_1 l_2 + K_2 l_1}$$

It is given that $K_{Cu} = 9K_s$. So, if $K_s = K_1 = K$, then

$$K_{Cu} = K_2 = 9K$$

$$\Rightarrow \quad \theta = \frac{9K \times 100 \times 6 + K \times 0 \times 18}{9K \times 6 + K \times 18}$$

$$= \frac{5400\,K}{72\,K} = 75°\text{C}$$

10. Equivalent thermal conductivity of the compound slab,

$$K_{eq} = \frac{l_1 + l_2}{\frac{l_1}{K_1} + \frac{l_2}{K_2}} = \frac{l + l}{\frac{l}{K} + \frac{l}{2K}}$$

$$= \frac{2l}{\frac{3l}{2K}} = \frac{4}{3}K$$

11. As we know, $Q \propto T^4$

$$\Rightarrow \quad \frac{H_A}{H_B} = \left[\frac{273 + 727}{273 + 327}\right]^4 = \frac{625}{81}$$

12. Total energy radiated from a body

$$Q = A\varepsilon\sigma T^4 t \quad \text{or} \quad \frac{Q}{t} \propto AT^4$$

$$\frac{Q}{t} \propto r^2 T^4 \qquad (\because A = 4\pi r^2)$$

$$\frac{Q_1}{Q_2} = \left(\frac{r_1}{r_2}\right)^2 \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{8}{2}\right)^2 \left[\frac{273 + 127}{273 + 527}\right]^4 = 1$$

13. According to Wien's displacement law,

$$\lambda_m T = b \quad \text{or} \quad \lambda_m \propto \frac{1}{T}$$

where, b is Wien's constant whose value is 2.9×10^{-3} mK.

$$\frac{(\lambda_m)_S}{(\lambda_m)_F} = \frac{T_F}{T_S}.$$

or $$T_F = T_S \times \frac{(\lambda_m)_S}{(\lambda_m)_F} = 5500\text{ K} \times \frac{(5.5 \times 10^{-7}\text{ m})}{(11 \times 10^{-7}\text{ m})} = 2750\text{ K}$$

14. An ideal black body absorbs all the radiations incident upon it and has an emissivity equal to 1. If a black body and an identical another body are kept at the same temperature, then the black body will radiate maximum power.

Hence, the black object at a temperature of 2000°C will glow brightest.

15. According to Newton's cooling law, option (c) is correct answer.

16. According to Newton's law of cooling, t_1 will be less than t_2.

17. We know that, $\frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A} = \frac{500}{1500} = \frac{1}{3}$

$E \propto T^4 A$ (where, A = surface area = $4\pi R^2$)

$$E \propto T^4 R^2$$

$$\frac{E_A}{E_B} = \left(\frac{T_A}{T_B}\right)^4 \left(\frac{R_A}{R_B}\right)^2 = (3)^4 \left(\frac{16}{18}\right)^2 = 9$$

18. According to Wien's displacement law,

$$\lambda_m T = \text{constant}$$

$$\therefore \quad \frac{(\lambda_m)_1}{(\lambda_m)_2} = \frac{T_2}{T_1}$$

Here, $\frac{T_1}{T_2} = \frac{3}{2}$, $(\lambda_m)_1 = 4000\,\text{Å} = 4000 \times 10^{-10}$ m

$$\therefore \quad (\lambda_m)_2 = \frac{4000 \times 10^{-10} \times 3}{2} = 6000\,\text{Å}$$

19. Luminosity of a star depends upon the total radiations emitted by the star.

The star emits 17000 times the radiations emitted by the sun.

$$\therefore \quad E = \sigma T^4$$

Hence, $\frac{E_1}{E} = \left(\frac{T_1}{T}\right)^4$

So, $(17000)^{1/4} = \frac{T_1}{T}$ (Given, $E_1 = 17000E$)

$$T_1 = 6000 \times 11.4 = 68400\text{ K}$$

20. According to Newton's law of cooling,

$$\frac{\theta_1 - \theta_2}{t} = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

***Case* I**

$$\Rightarrow \quad \frac{80 - 64}{5} = K\left[\frac{80 + 64}{2} - \theta_0\right]$$

$$\Rightarrow \quad 3.2 = K[72 - \theta_0] \quad \ldots\text{(i)}$$

***Case* II**

$$\Rightarrow \quad \frac{64 - 52}{5} = K\left[\frac{64 + 52}{2} - \theta_0\right]$$

$$2.4 = K[58 - \theta_0] \quad \ldots\text{(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{3.2}{2.4} = \frac{72 - \theta_0}{58 - \theta_0}$$

$$185.6 - 3.2\,\theta_0 = 172.8 - 2.4\,\theta_0$$

$$\Rightarrow \quad \theta_0 = 16°\text{C}$$

[CHEMISTRY]

21. Lower the pK_a, stronger is the acid.

22.

$$\begin{array}{lccc} & HQ & \rightleftharpoons H^+ & + Q^- \\ \text{At } t=0 & 0.1 & - & - \\ \text{At Eqm.} & (0.1-x) & x & x \end{array}$$

$$pH = 3 \Rightarrow [H^+] = x = 10^{-3}$$

$$0.1 - x \simeq 0.1$$

$$K_a = \frac{x \cdot x}{0.1 - x} = \frac{10^{-3} \times 10^{-3}}{0.1} = \mathbf{10^{-5}}$$

Alternatively :

$$x = c\alpha = 10^{-3}$$

$$\alpha = \frac{10^{-3}}{0.1} = 10^{-2}$$

$$K_a = c\alpha^2 = 0.1 \times 10^{-4} = \mathbf{10^{-5}}$$

23. $[OH^-] = \frac{100 \times 0.2 - 100 \times 0.1}{1000} = 10^{-2}$

$\Rightarrow \quad pOH = 2 \Rightarrow pH = \mathbf{12}$

24. Change of pH from 1 to 2 $\Rightarrow$ change in $[H^+]$ from 10^{-1} M to 10^{-2} M

$$M_2V_2 (\text{dil.}) = M_1V_1 (\text{conc.})$$

$$V_2 = \frac{10^{-1} \times 1}{10^{-2}} = 10\,L$$

Volume of H_2O added = 10 – 1 = **9L**

25.

CH_3COOH is a weak acid. $10^{-2}M$ CH_3COOH will give much less $[H^+]$ concentration than 10^{-2} M. Hence, pH will be **more than 2.**

26.

$$pH = 3 \Rightarrow [H^+] = 10^{-3} M$$

On dilution, $[H^+] = \frac{1}{2} \times 10^{-3} = 5 \times 10^{-4} M$

New $pH = -\log(5 \times 10^{-4}) = -(0.699 - 4) = \mathbf{3.301}$

27.

$$\underset{}{MX_4} \rightleftharpoons \underset{S}{M} + \underset{4S}{4X}$$ (S is solubility in mol L^{-1})

$$K_{sp} = (S)(4S)^4 = 256 S^5$$

$$\boldsymbol{S = \left[\frac{K_{sp}}{256}\right]^{1/5}}$$

28.

$$MX_2 \rightleftharpoons \underset{S}{M^{2+}} + \underset{2S}{2X^-}$$ (S is solubility in mol L^{-1})

$$K_{sp} = S \times (2S)^2 = 4S^3$$

$$4S^3 = 4 \times 10^{-12}$$

$$S = \mathbf{1 \times 10^{-4}\ M}$$

29.

K_{sp} of $Cr(OH)_3 = S \times 3^3 S^3$

$$27S^4 = 1.6 \times 10^{-30}$$

$$S = \sqrt[4]{\mathbf{1.6 \times 10^{-30} / 27}}$$

30.

$A_3B_2 \rightleftharpoons \underset{3x}{3A} + \underset{2x}{2B}$ [for solubility of A_3B_2 as xM]

$$K_{sp} = [A]^3 \times [B]^2 = [3x]^3 \times [2x]^2 = \mathbf{108\,x^5}$$

31.

Solubility is directly proportional to K_{sp}. MnS has highest K_{sp} among the given substances and hence has highest solubility.

32.

$CaF_2 \rightleftharpoons Ca^{2+} + 2F^-$

For solubility 'S', $K_{sp} = (S)(2S)^2 = 4S^3$ (S is solubility)

$$4S^3 = 3.2 \times 10^{-11}$$

$$S^3 = 8 \times 10^{-12}$$

$$S = \mathbf{2 \times 10^{-4}\ M}$$

33.

For solubility S, K_{sp} of $A_2B_3 = (2)^2 \times (3)^3 \times S^2 \times S^3 = 108 \times (1 \times 10^{-2})^5$

$$= 108 \times 10^{-10} = \mathbf{1.08 \times 10^{-8}}$$

34.

$HgCl_2 \rightleftharpoons Hg^{2+} + 2Cl^-$

$K_{sp} = S \times (2S)^2 = 4S^3$ (S is solubility)

$$4S^3 = 4 \times 10^{-15}$$

$$S = 10^{-5}$$

$$[Cl^-] = 2S = \mathbf{2 \times 10^{-5}\ M}$$

35.

Higher the K_{sp}, higher is the solubility.

36.

$PbCl_2 \rightleftharpoons \underset{s}{Pb^{2+}} + \underset{2s}{2Cl^-}$

For solubility 's'

$$K_{sp} = 4s^3 \quad \Rightarrow \quad s = \left(\frac{1}{4} \times 10^{-6}\right)^{\frac{1}{3}}$$

$$= \mathbf{(0.25 \times 10^{-6})^{1/3}}$$

37.

If 1 L of each solution is mixed,

$$[H^+] = \frac{10^{-3} + 10^{-4} + 10^{-5}}{3}$$

$$= \frac{1.11 \times 10^{-4}}{3} = \mathbf{3.7 \times 10^{-4}\ M}$$

38.

$$AgCl \rightleftharpoons \underset{s}{Ag^+} + \underset{s}{Cl^-}$$
For solubility 's'

$$NaCl \longrightarrow \underset{0.1}{Na^+} + \underset{0.1}{Cl^-}$$

$$[Ag^+]+[Cl^-] = K_{sp} \Rightarrow (s)(0.1) = 1.2\times10^{-10}$$

$$s = \mathbf{1.2\times10^{-9}}\ M$$

39.

40.

$$C\alpha^2 = K_a$$

$$[H^+] = C\alpha = \frac{K_a}{\alpha}$$

$$pH = -\log\frac{K_a}{\alpha}$$

$$= -\log K_a + \log\alpha$$

$$= -\log 10^{-9} + \log\left(\frac{0.01}{100}\right)$$

$$= +9-4 = \mathbf{5}$$

[MATHEMATICS]

41. (b) $\frac{2b^2}{a} = b \Rightarrow \frac{b}{a} = \frac{1}{2} \Rightarrow \frac{b^2}{a^2} = \frac{1}{4}$

Hence $e = \sqrt{1-\frac{b^2}{a^2}} = \frac{\sqrt{3}}{2}$.

42 (b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Since it passes through (–3, 1) and (2, –2), so $\frac{9}{a^2}+\frac{1}{b^2}=1$ and $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{4} \Rightarrow a^2=\frac{32}{3}$, $b^2=\frac{32}{5}$

Hence required equation of ellipse is $3x^2+5y^2=32$.

Trick : Since only equation $3x^2+5y^2=32$ passes through (–3, 1) and (2, –2). Hence the result.

43 (a) Given $\frac{2b^2}{a}=10$ and $2b=2ae$

Also $b^2=a^2(1-e^2) \Rightarrow e^2=(1-e^2) \Rightarrow e=\frac{1}{\sqrt{2}}$

$\Rightarrow b=\frac{a}{\sqrt{2}}$ or $b=5\sqrt{2}$, $a=10$

Hence equation of ellipse is $\frac{x^2}{(10)^2}+\frac{y^2}{(5\sqrt{2})^2}=1$

i.e., $x^2+2y^2=100$.

44. (d) $e=\frac{1}{\sqrt{2}}$; Latus rectum $=\frac{2b^2}{a}=\frac{2a^2}{a}\left(1-\frac{1}{2}\right)=a$

i.e., semi-major axis.

45. (a) Let the equation of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\because$ It passes through (– 3, 1)

So, $\frac{9}{a^2}+\frac{1}{b^2}=1 \Rightarrow 9+\frac{a^2}{b^2}=a^2$ .....(i)

Given eccentricity is $\sqrt{2/5}$

So, $\frac{2}{5}=1-\frac{b^2}{a^2} \Rightarrow \frac{b^2}{a^2}=\frac{3}{5}$ .....(ii)

From equation (i) and (ii), $a^2=\frac{32}{3}, b^2=\frac{32}{5}$

Hence required equation of ellipse is $3x^2+5y^2=32$.

46. (b) $\frac{x^2}{2-r}+\frac{y^2}{r-5}+1=0 \Rightarrow \frac{x^2}{r-2}+\frac{y^2}{5-r}=1$

Hence $r>2$ and $r<5 \Rightarrow 2<r<5$.

47. (a) The ellipse is $4(x-1)^2+9(y-2)^2=36$

Therefore, latus rectum $=\frac{2b^2}{a}=\frac{2.4}{3}=\frac{8}{3}$.

48. (b) $4x^2-8x+y^2+2y+1=0$

$\Rightarrow (2x-2)^2+(y+1)^2=-1+4+1$

$\Rightarrow \frac{(x-1)^2}{1}+\frac{(y+1)^2}{4}=1 \Rightarrow e=\sqrt{1-\frac{1}{4}} \Rightarrow e=\frac{\sqrt{3}}{2}$.

49. (a) Let any point on it be (x,y), then $\frac{\sqrt{(x+1)^2}+\sqrt{(y-1)^2}}{\left|\frac{x-y+3}{\sqrt{2}}\right|}=\frac{1}{2}$

Squaring and simplifying, we get

$7x^2+2xy+7y^2+10x-10y+7=0$.

50. (b) $c=\pm\sqrt{b^2+a^2m^2}=\pm\sqrt{4+8.4}=\pm 6$.

51. (b) $\frac{2b^2}{a}=b \Rightarrow \frac{b}{a}=\frac{1}{2} \Rightarrow \frac{b^2}{a^2}=\frac{1}{4}$

Hence $e=\sqrt{1-\frac{b^2}{a^2}}=\frac{\sqrt{3}}{2}$.

52. (b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Since it passes through (–3, 1) and (2, –2),

so $\frac{9}{a^2}+\frac{1}{b^2}=1$ and $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{4} \Rightarrow a^2=\frac{32}{3}$, $b^2=\frac{32}{5}$

Hence required equation of ellipse is $3x^2+5y^2=32$.

Trick : Since only equation $3x^2+5y^2=32$ passes through (–3, 1) and (2, –2). Hence the result.

53. (c) $\frac{x^2}{\frac{112}{16}}+\frac{y^2}{\frac{112}{7}}=1$. Therefore, $e=\sqrt{1-\frac{112}{16}.\frac{7}{112}}=\frac{3}{4}$.

54. (b) Here given that $2b = 10, 2a = 8 \Rightarrow b = 5,\ a = 4$

Hence the required equation is $\frac{x^2}{16} + \frac{y^2}{25} = 1$.

55. (c) Let point be (h, k) their pair of tangent will be

$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) = \left(\frac{hx}{a^2} + \frac{yk}{b^2} - 1\right)^2$$

Pair of tangents will be perpendicular, if coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow \frac{k^2}{a^2b^2} + \frac{h^2}{a^2b^2} = \frac{1}{a^2} + \frac{1}{b^2} \Rightarrow h^2 + k^2 = a^2 + b^2$$

Replace (h, k) by $(x, y) \Rightarrow x^2 + y^2 = a^2 + b^2$.

56. (c) Focal distance of any point $P\ (x,y)$ on the ellipse is equal to $SP = a + ex$. Here $x = a\cos\theta$

Here $SP = a + ae\cos\theta = a(1 + e\cos\theta)$.

57. (a) Let point $P\ (x_1, y_1)$

So, $\sqrt{(x_1 + 2)^2 + y_1^2} = \frac{2}{3}\left(x_1 + \frac{9}{2}\right)$

$\Rightarrow (x_1 + 2)^2 + y_1^2 = \frac{4}{9}\left(x_1 + \frac{9}{2}\right)^2$

$\Rightarrow 9[x_1^2 + y_1^2 + 4x_1 + 4] = 4\left(x_1^2 + \frac{81}{4} + 9x_1\right)$

$\Rightarrow 5x_1^2 + 9y_1^2 = 45 \Rightarrow \frac{x_1^2}{9} + \frac{y_1^2}{5} = 1$,

Locus of (x_1, y_1) is $\frac{x^2}{9} + \frac{y^2}{5} = 1$, which is equation of an ellipse.

58. (b) $SP + S'P = 2a = 2.6 = 12$.

59. (c) In the first case, eccentricity $e = \sqrt{1 - (25/169)}$

In the second case, $e' = \sqrt{1 - (b^2/a^2)}$

According to the given condition,

$\sqrt{1 - b^2/a^2} = \sqrt{1 - (25/169)}$

$\Rightarrow b/a = 5/13$, $(\because a > 0, b > 0)$

$\Rightarrow a/b = 13/5$.

60. (b) Foci $= (3,-3) \Rightarrow ae = 3 - 2 = 1$

Vertex $= (4,-3) \Rightarrow a = 4 - 2 = 2 \Rightarrow e = \frac{1}{2}$

$\Rightarrow b = a\sqrt{\left(1 - \frac{1}{4}\right)} = \frac{2}{2}\sqrt{3} = \sqrt{3}$

Therefore, equation of ellipse with centre $(2,-3)$ is

$\frac{(x-2)^2}{4} + \frac{(y+3)^2}{3} = 1$.